SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2009 SECOND ROUND
SENIOR SECTION: GRADES 10, 11 AND 12

21 May 2009 **Time: 120 minutes** **Number of questions: 20**

Instructions

1. *Do not open this booklet until told to do so by the invigilator.*
2. *This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.*
3. *Scoring rules:*
   3.1. *Each correct answer is worth 4 marks in part A, 5 marks in part B and 6 marks in part C.*
   3.2. *For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.*
4. *You must use an HB pencil. Rough paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
5. *Diagrams are not necessarily drawn to scale.*
6. *Indicate your answers on the sheet provided.*
7. *Start when the invigilator tells you to do so. You have 120 minutes to complete the question paper.*
8. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so
Draai die boekie om vir die Afrikaanse vraestel

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9323 E-mail: ellie@samf.ac.za

Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

PRACTICE EXAMPLES

1. As a decimal number 6.28% is equal to

(A) 0.0628 (B) 0.628 (C) 6.28 (D) 62.8 (E) 628

2. The value of $1+\dfrac{1}{3+\dfrac{1}{2}}$ is

(A) $\dfrac{6}{5}$ (B) $\dfrac{7}{6}$ (C) $\dfrac{9}{2}$ (D) $\dfrac{6}{7}$ (E) $\dfrac{9}{7}$

3. The tens digit of the product $1 \times 2 \times 3 \times \cdots \times 98 \times 99$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

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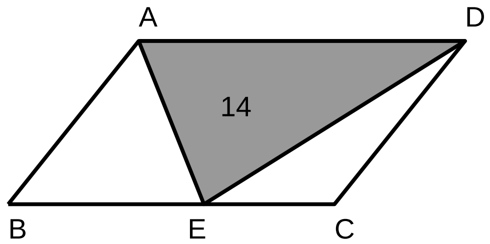
Part A: Four marks each

1. What is the sum of five consecutive integers if the middle one is n?

(A) $n+5$ (B) n^5 (C) $5n$ (D) $n+1$ (E) $5n+5$

2. ABCD is a parallelogram and E is a point on BC. If the area of triangle AED is 14, then the area of ABCD is



(A) impossible to determine (B) 24 (C) 25 (D) 27 (E) 28

3. If $\dfrac{a}{2} = \dfrac{b}{3} = \dfrac{c}{5}$ and $abc = 810$, then the value of b is

(A) 6 (B) 12 (C) 9 (D) 18 (E) 15

4. Nico had an average mark of 85 for his first eight tests, and an average of 81 for the first nine. What was his mark for the ninth test?

(A) 49 (B) 53 (C) 61 (D) 68 (E) 74

5. The operation $\langle x \rangle$ satisfies

(i) $\langle 1 \rangle = 1$

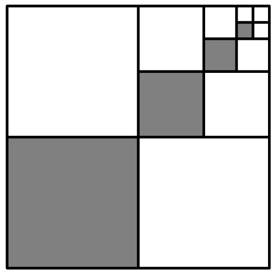
(ii) $\langle x + y \rangle = \langle x \rangle + \langle y \rangle + x\,y$

for all positive integers x and y. The value of $\langle 3 \rangle$ is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 8

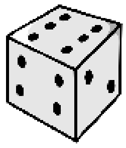
Part B: Five marks each

6. The pattern of shading one quarter of a square is shown in the diagram. If this pattern is continued indefinitely, what fraction of the large square will eventually be shaded?



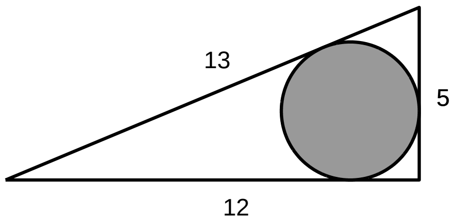
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{2}{7}$ (E) $\frac{3}{8}$

7. If a dice is rolled twice, the probability that the second number is greater than the first is



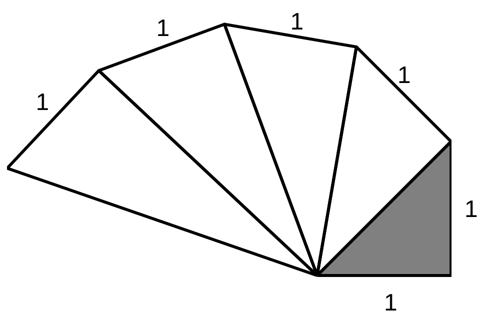
(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$ (E) $\frac{5}{18}$

8. The radius of the largest circle that can fit into a triangle with sides 5, 12, 13, is



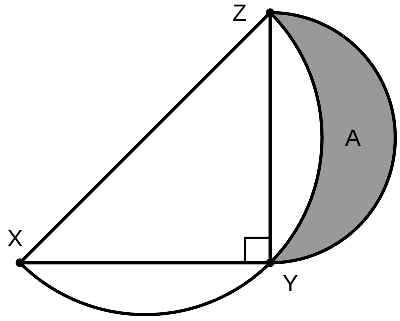
(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{5}$ (E) $\sqrt{6}$

9. A sequence of right-angled triangles is drawn, starting with the shaded one. The area of the 100 th triangle is



(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $3\sqrt{3}$ (D) 5 (E) 6

10. ΔXYZ is an isosceles right-angled triangle, with XY $=$ YZ $= 1$. Two semi-circles are drawn, one with diameter XZ and the other with diameter YZ. The shaded area A is equal to

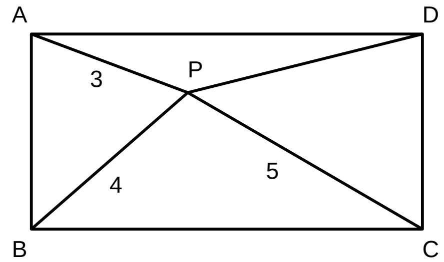


(A) $\dfrac{1}{4}$ (B) $\dfrac{\pi}{12}$ (C) $\dfrac{1}{3}$ (D) $2\pi - 6$ (E) $1 - \dfrac{1}{4}\pi$

11. $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \dfrac{1}{\sqrt{2008}+\sqrt{2009}}$ equals

(A) $\dfrac{1}{\sqrt{2009}-\sqrt{2008}}$ (B) $\sqrt{2009}-1$ (C) $\sqrt{2010}$ (D) $\sqrt{2009}+1$ (E) $\sqrt{2008}+1$

12. P is a point inside rectangle ABCD.
If AP $= 3$, BP $= 4$ and CP $= 5$, then DP is



(A) impossible to determine (B) $3\sqrt{2}$ (C) $2\sqrt{3}$ (D) $2\sqrt{5}$ (E) 4

13. For every real number x, let $m(x)$ be the smallest of the three numbers $x+1$, $\dfrac{x}{2}+2$ and $-x+8$. What is the largest value of $m(x)$?

(A) 2 (B) 4 (C) 1 (D) 3 (E) 5

14. If n is a positive integer, what is the smallest value of n such that $(n+20)+(n+21)+(n+22)+ \cdots +(n+100)$ is a perfect square?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

15. If x and y are real numbers, then the minimum value of $x^2 - 2xy + 2y^2 - 6y$ is

(A) -12 (B) -11 (C) -10 (D) -9 (E) -8

Part C: Six marks each

16. If the product

$$\frac{5}{9}\cdot\frac{12}{16}\cdot\frac{21}{25}\cdot\frac{32}{36}\cdot\cdots$$

is continued indefinitely, then eventually its value becomes

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{1}{6}$

17. The largest prime factor of $93!+94!+95!$ is
(Note: $n! = 1\times 2\times 3\times\cdots\times n$)

(A) 83 (B) 89 (C) 91 (D) 97 (E) 101

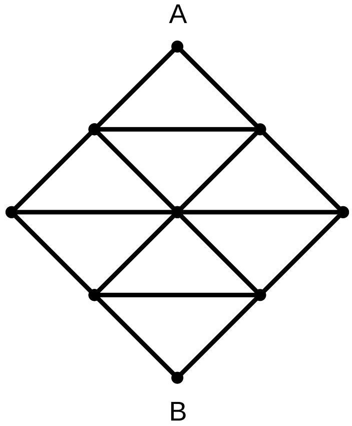
18. The smallest integer n such that $\sqrt{n}-\sqrt{n-1} < 0.01$ is

(A) 2500 (B) 2335 (C) 2501 (D) 2336 (E) 2233

19. Let x be the ten-digit number 9 999 999 999. The sum of the digits of x^3 is

(A) 99 (B) 108 (C) 180 (D) 199 (E) 297

20. You have to move from point A to point B by a path that goes either downwards or horizontally along the line segments in the diagram. You may not use any segment more than once. How many different paths are possible?



(A) 24 (B) 64 (C) 96 (D) 192 (E) 48