THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION
Sponsored by HARMONY GOLD MINING

FIRST ROUND 2007

SENIOR SECTION: GRADES 10, 11 AND 12

20 MARCH 2007

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 5 marks.
   - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 60 minutes to complete the question paper.
8. Answers and solutions are available at: http://www.samf.ac.za/

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0.5129; 0.9; 0.89; and 0.289 is

(A) 1.189
(B) 0.8019
(C) 1.428
(D) 1.179
(E) 1.4129

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1. If biltong costs R16 for every 80 grams, then the cost per kilogram is

(A) R80 (B) R20 (C) R50 (D) R200 (E) R160

2. Eighty people stand in a queue to collect sandwiches. There are enough sandwiches for each person to have three sandwiches. Instead, every person at the front of the queue takes five sandwiches until there are none left. The number of people who do not get any sandwiches is

(A) 40 (B) 32 (C) 20 (D) 50 (E) 24

3. In a mathematics class the learner attendance doubles every school day. If the class is full on Thursday, 18 January 2007, on which date in January was the class half full?

(A) 10 (B) 15 (C) 9 (D) 16 (E) 17

4. Full cream milk has about 3.4 g of fat per 100 ml and "fat free" milk has about 0.5 g of fat per 100 ml. The approximate percentage of the fat that is removed from full cream milk to produce "fat free" milk, is

(A) 7 (B) 17 (C) 71 (D) 85 (E) 97

5. If $a_0 = 2, a_1 = 0$ and $a_n = 2a_{n-1} + a_{n-2}$, $n = 2, 3, 4, \ldots$, then a_5 equals

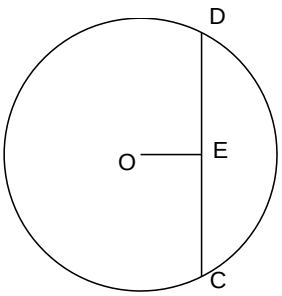
(A) 19 (B) 18 (C) 25 (D) 21 (E) 24

6. The figure shows a 3×3 magic square made from the first 9 natural numbers, in which each row and each column adds to the same number. A 5×5 magic square is made from the first 25 natural numbers. The sum of each row and each column of the 5×5 magic square is

8	1	6
3	5	7
4	9	2

(A) 60 (B) 50 (C) 70 (D) 65 (E) 55

7. In the figure O is the centre of the circle, $OE = 2, CD = 8$, and OE is perpendicular to CD. The circumference of the circle is



(A) 16π (B) 25π (C) $\sqrt{5}\pi$ (D) $2\sqrt{5}\pi$ (E) $4\sqrt{5}\pi$

8. The number of solutions of the equation $\sqrt{x^4 + 16} = x^2 + 4$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) infinite

9. Twenty numbers have an average of 20. Nine of these numbers have an average of 9. The average of the remaining eleven numbers is

(A) 20 (B) 31 (C) 21 (D) 11 (E) 29

10. The sequence 3; 9; 7; 13; 11; 17; 15; x; 19; 25 follows a fixed pattern. The value of x is

(A) 21 (B) 20 (C) 18 (D) 22 (E) 23

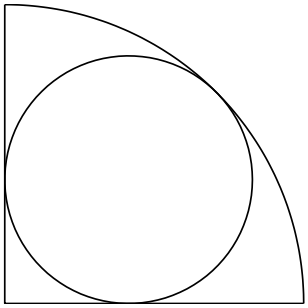
11. On a flat part of the earth a monorail is built between two towns 20 km apart, using one metal rail. By accident the bar is made one metre too long and the contractor decides to lift it in the middle in order make the ends fit. Approximately how high does she have to lift it in the middle?

(A) 1 cm (B) 1 m (C) 10 m (D) 100 m (E) 1 km

12. Which one of following fractions is the smallest?

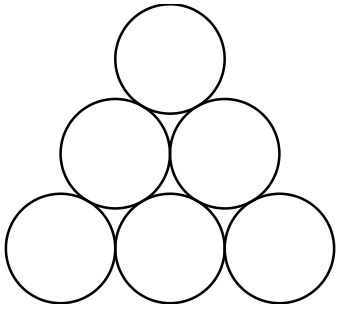
(A) $\dfrac{20052005}{20052006}$ (B) $\dfrac{20072007}{20072008}$ (C) $\dfrac{20042004}{20042005}$ (D) $\dfrac{20062006}{20062007}$ (E) $\dfrac{20092009}{20092010}$

13. A circle is inscribed in a quarter of a circle with radius 8. The radius of the smaller circle is



(A) $\frac{4}{1+\sqrt{2}}$ (B) 4 (C) $2\sqrt{2}$ (D) $\frac{8}{1+\sqrt{2}}$ (E) $1+2\sqrt{2}$

14. Each of the circles in the figure has radius 5. The height of the figure is

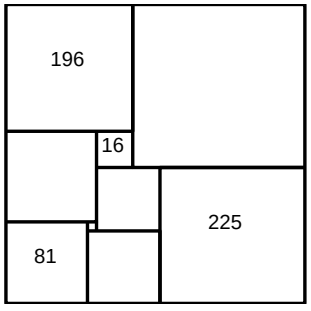


(A) 15 (B) $10\sqrt{3}+10$ (C) $10\sqrt{3}$ (D) $15\sqrt{3}+10$ (E) $20\sqrt{3}+10$

15. A daughter writes down her own age directly after her mother's, forming a four-digit number. From this four-digit number she subtracts the difference between her mother's age and her age to get 4202. The age of the daughter is

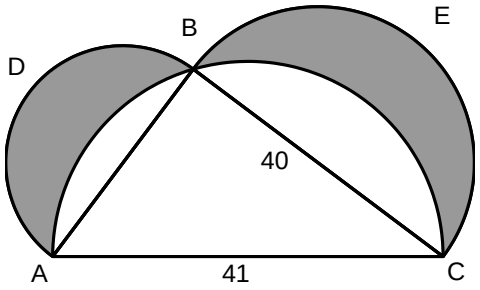
(A) 42 (B) 12 (C) 32 (D) 22 (E) 26

16. The rectangle shown in the figure is divided into squares of different sizes, with areas as shown. The area of the whole rectangle is



(A) 1024 (B) 1056 (C) 1089 (D) 1120 (E) 1122

17. In the figure AC is the diameter of the semicircle ABC. AB is the diameter of the semicircle ADB and BC is the diameter of the semicircle BEC. If $BC = 40$ and $AC = 41$ then the area of the shaded region is



(A) 90 (B) 180 (C) 60 (D) 150 (E) 120

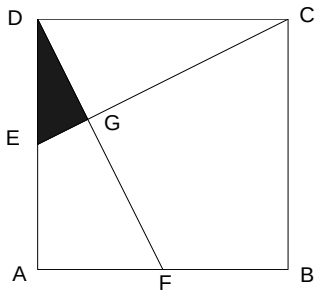
18. If R is the remainder when each of the numbers 128, 227, and 73 is divided by D, where D is an integer greater than 1, then $D - R$ equals

(A) 5 (B) 11 (C) 7 (D) 13 (E) 4

19. A large solid cube is built of identical smaller cubes such that no more than half the small cubes are not visible from outside. The least number of small cubes that can be used to build the large cube is

(A) 1728 (B) 125 (C) 729 (D) 1000 (E) 1331

20. $ABCD$ is a square, E and F are the midpoints of AD and AB as shown. If the area of triangle DEG is 1, then the area of $ABCD$ is



(A) 14 (B) 12 (C) 16 (D) 18 (E) 20