HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

SAMF
SOUTH AFRICAN MATHEMATICS FOUNDATION

HARMONY™

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

THIRD ROUND 2010
JUNIOR SECTION: GRADES 8 AND 9

8 SEPTEMBER 2010
TIME: 4 HOURS
NUMBER OF QUESTIONS: 15
TOTAL: 100

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- For Question 9 you need a strip of paper that will be in your answer book.
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on one question.
- Answers and solutions are available at: www.samf.ac.za

DO NOT TURN THE PAGE

UNTIL YOU ARE TOLD TO DO SO.

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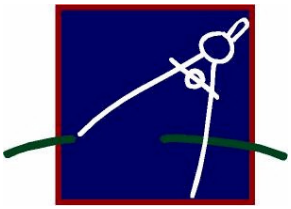
Organizations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns



Question 1

One of the following five numbers is the average of the other four.

Which one is it?

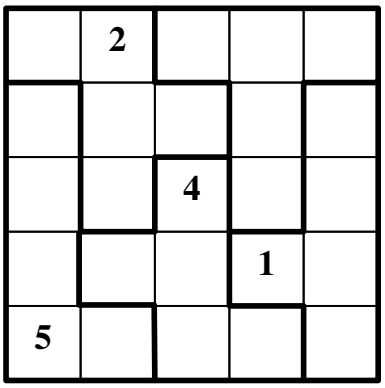
26 ; 30 ; 37 ; 28 ; 29

[4]

Question 2

Place a single digit in each empty square in the diagram so that each row, each column and each jigsaw piece contains all numbers from 1 to 5.

(Fill in the numbers on the answer sheet)



[4]

Question 3

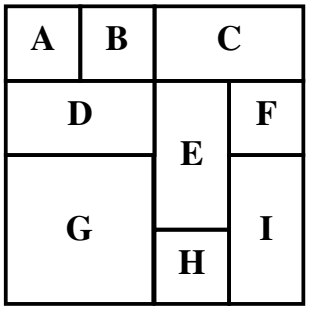
Find the size of the smaller angle between the hands of a clock when the time is 19:30.

[6]

Question 4

Nine 2 x 2 squares of paper, each labelled with a different letter, are placed on a table after each other, resulting in the 4 by 4 square shown.

In what order were the 9 squares of paper placed from first to last?



[6]

Question 5

(a) Find 28 consecutive integers that add up to 294.

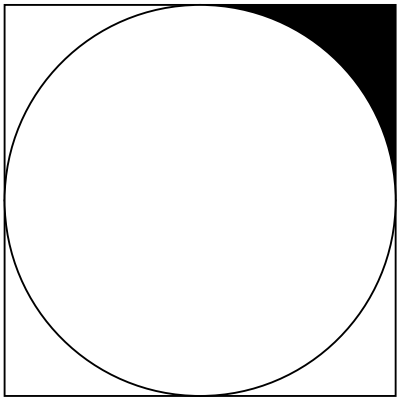
(b) Prove that it is not possible to find 3 consecutive integers that add up to 50.

[6]

Question 6

(a) Is the statement $\pi = \frac{22}{7}$ true or false? Why?

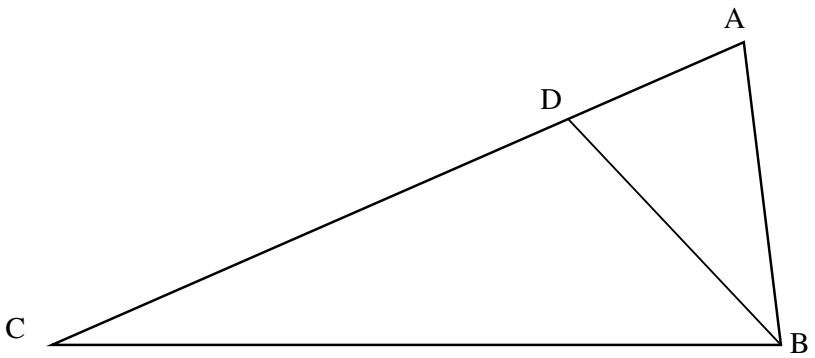
(b) The figure shows a circle drawn into a square.
If the perimeter of the shaded region is 25 cm,
estimate the area of the circle taking π to be $\frac{22}{7}$.



[6]

Question 7

In ΔABC point D is on AC,
AB = AD,
and $\hat{ABC} - \hat{ACB} = 30^\circ$.
Find $\hat{CBD}$.



[6]

Question 8

Find the smallest positive integer K which, when successively divided by 6 ; 5 ; 4 ; 3 ; 2, leaves remainders of 5 ; 4 ; 3 ; 2 ; 1 respectively.

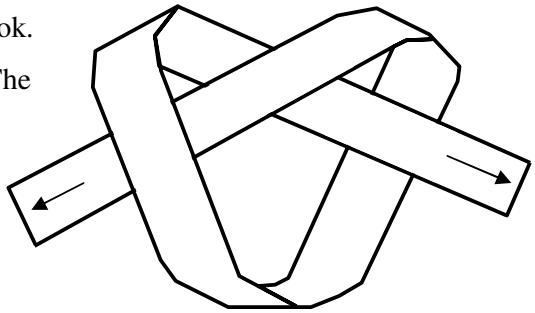
[6]

Question 9

You have been given 2 strips of paper in your answer book.
Tie a knot into one of the strips of paper and pull tight (The second strip is just if you don't get it right the first time)

(a) What shape do you get? Cool hey!!

(b) You will see that in some places your shape is 2 layers, 3 layers or 4 layers thick.
Describe the shape that is 4 layers thick.



[8]

Question 10

Consider the following pattern.

Row 1: $1 + 2 = 3$

Row 2: $4 + 5 + 6 = 7 + 8$

Row 3: $9 + 10 + 11 + 12 = 13 + 14 + 15$

Row …

…

…

Row *n*:

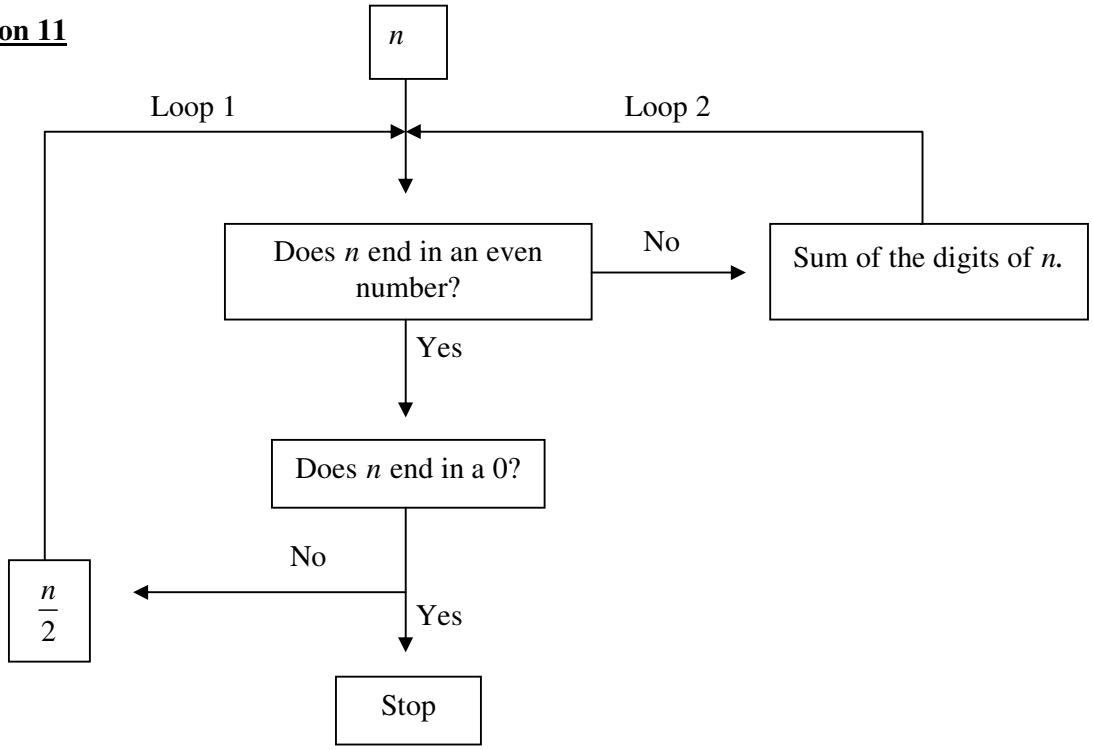
(a) Give a formula, in terms of *n*, for the last term on the right-hand side of row *n*.

(b) Give a formula, in terms of *n*, for the last term on the left-hand side of row *n*.

(c) Find a formula for the sum of either side of the equation of row *n*.

[8]

Question 11



(a) If $n = 49\,997$ how many times does it go through each loop and what is the final output?

(b) Some positive even numbers go through both loops at least once. Describe these numbers.

(c) Some positive odd numbers less than 150 go through both loops at least once. Describe these numbers.

[8]

Question 12

Before his last Maths test, Bongani's average for Maths was 33%. In his last test he scored 40% which increased his average to 34%.
What must he score in his next test to increase his average to 35%?

[8]

Question 13

Fred puts 11 plastic bags inside another plastic bag. Each of the 11 bags is either empty or itself contains another 11 bags. All together 6 bags contain other bags.
Of all the bags, how many remain empty?

[8]

Question 14

On a far-away planet in a far-away galaxy the people have 13 fingers and so they count a little differently – they have three extra letters: *x, y, z*.

Our numbers: 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9 ; 10 ; 11 ; 12 ; 13 ; 14

Their numbers: 1 ; 2 ; 3 ; *x* ; 4 ; 5 ; *y* ; 6 ; 7 ; 8 ; *z* ; 9 ; 10 ; 11

Our 20 is represented by $1y$ and our 100 by $y7$ and so on.
What will the square of $1x$ be written like on that far-away planet?

[8]

Question 15

Mr Mahlanyana had 9 children and 31 grandchildren. In his last will and testament he left an amount of money to each grandchild. Each girl was to get R7 more than each boy. All 31 grandchildren were alive when Mr Mahlanyana died and their legacies totaled R470. Of this amount R74 went to Mrs Zweni's children (she was Mr Mahlanyana's eldest daughter).
How many daughters did Mrs Zweni have?

[8]

Total: 100

THE END

Please turn over for the answer sheet for questions 2

ANSWER SHEET

Please hand in together with your answer booklet.

Name: _____ **School:** _____ **Grade:** _____

Question 2