HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

SAMF

SOUTH AFRICAN MATHEMATICS FOUNDATION

HARMONY™

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

THIRD ROUND 2009
JUNIOR SECTION: GRADES 8 AND 9

8 SEPTEMBER 2009
TIME: 4 HOURS
NUMBER OF QUESTIONS: 15

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- Questions 4, 5 and 11 should be done on the Answer Sheet provided (Please remember to write your Name and School on the answer sheet)
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Use your time wisely and do not spend all your time on one question.
- Answers and solutions are available at: www.samf.ac.za

DO NOT TURN THE PAGE

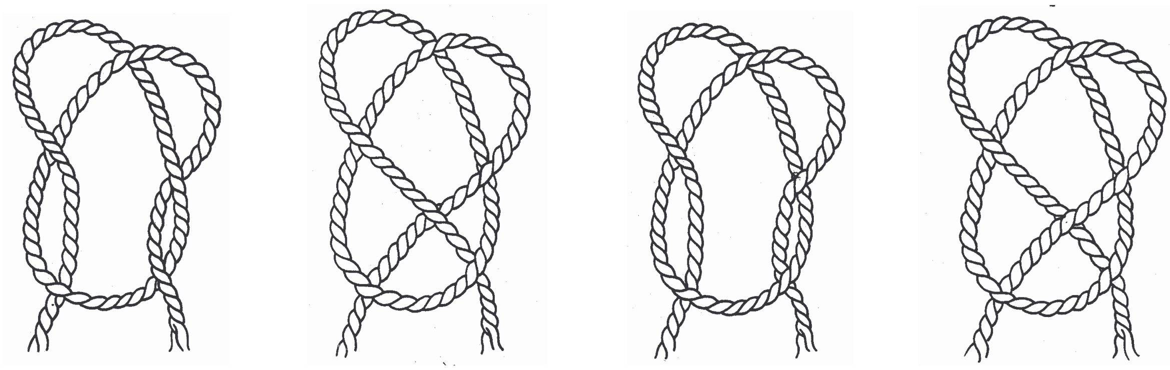
UNTIL YOU ARE TOLD TO DO SO.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9323 FAX: (012) 320-1950
E-mail:ellie@samf.ac.za

Organizations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns

Question 1

A B C D



Grasp the two loose ends of each rope firmly in your mind. Then imagine yourself pulling them until you have a straight piece of rope – either with a knot or without one.

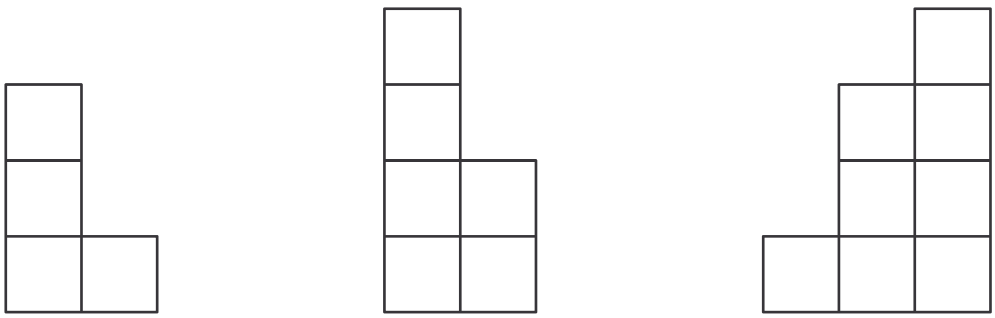
Which of these four ropes will give you a knot?

[4]

Question 2

A structure is built with identical cubes. The top view, the front view and the side view are shown below.

What is the least number of cubes required to build this structure?



Top view Front view Side view

[4]

Question 3

The digits of a two-digit number AB are reversed to give the number BA. These two numbers are added. For what values of A and B will the sum be a square number?

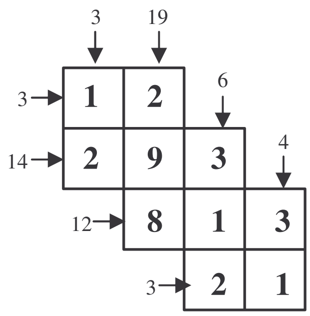
[6]

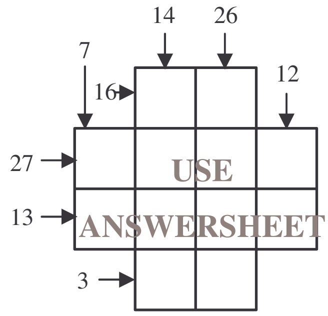
Question 4

Much like crossword puzzles, there are also **Cross Number puzzles** in which numbers from 1 to 9 need to be filled into the blocks in such a way that each vertical column and horizontal row adds up to the number shown above that column or to the left of that row. A number may not be repeated in any row or column.
An example is given.

Now complete this Cross Number puzzle.
(Fill in the numbers on the answer sheet)

Example



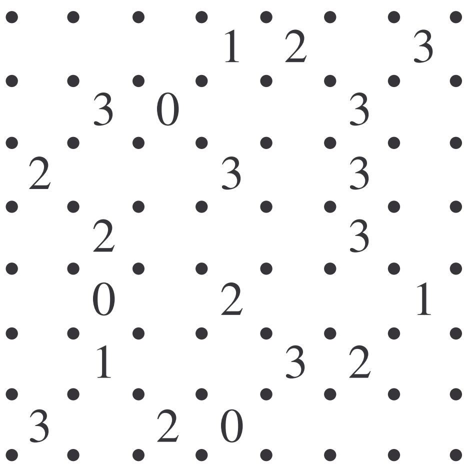


[6]

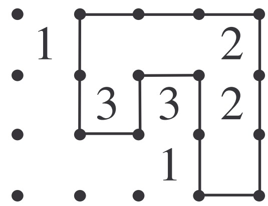
Question 5

Connect adjacent dots with vertical or horizontal lines so that a single closed loop is formed with no crossings or branches. Each number indicates how many lines surround it, while empty cells may be surrounded by any number of lines.
One correct and three incorrect examples are given.

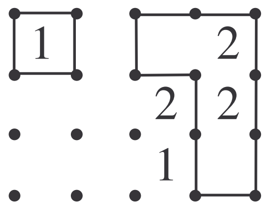
Use the Answer Sheet to complete this question.



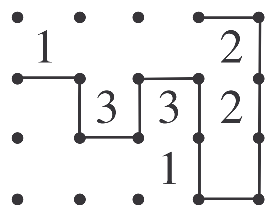
Correct Example



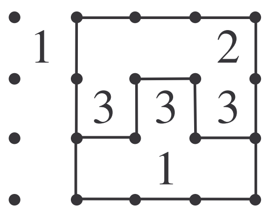
Incorrect Examples



(Two closed loops)



(Not a closed loop)



(Has branches)

[6]

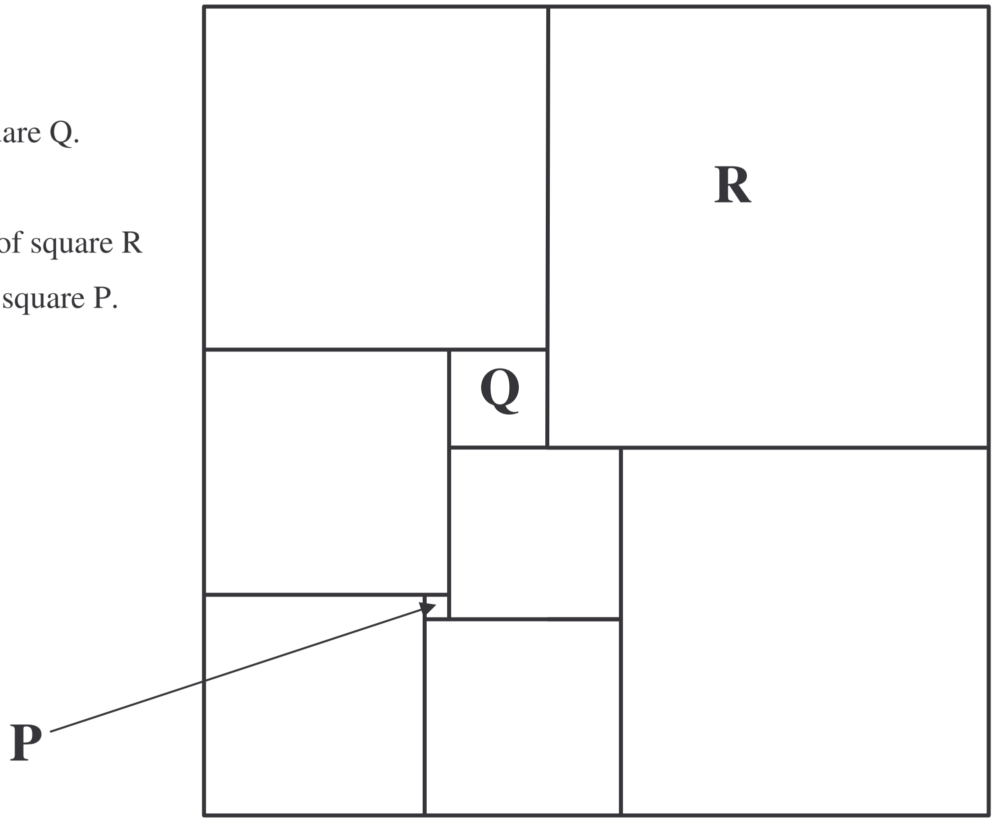
Question 6

Nine squares are arranged to form a rectangle as shown in the diagram.

Square P has an area of 1.

a) Find the area of square Q.

b) Prove that the area of square R is 324 times that of square P.



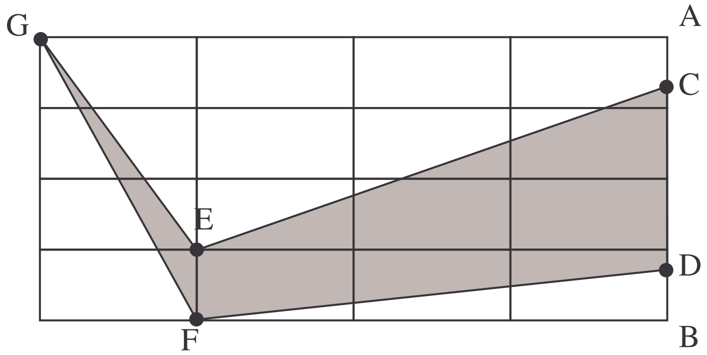
[6]

Question 7

In the figure the small rectangles are identical and each has an area of 8 cm^2.

C and D are points on the line segment AB as shown.

If $\text{CD} = \frac{2}{3}\text{AB}$, find the shaded area in cm^2.



[6]

Question 8

On a distant planet, railway tracks are built using one solid railway bar. A railway is built between two towns 20 km apart on a big flat section of the planet. Unfortunately the bar was made one metre too long and the constructor decided to lift it in the middle to try to make the ends fit.

Approximately how high does he have to lift it in the middle?
Is it 1 cm, 10 cm, 1 m, 10 m, 100 m or 1 km?

a) Guess one of the above, without doing any calculations. (1)

b) Calculate the answer and comment on how it compares with your guess. (5)

[6]

Question 9

Two candles of the same height are lit at the same time. The first candle is completely burnt up in 3 hours while the second candle is completely burnt up in 4 hours. At what point in time is the height of the second candle equal to twice that of the first candle?

[8]

Question 10

Nick and John play the following game. They put 100 pebbles on the table. During any move, a player takes at least one and not more than eight pebbles. Nick makes the first move, then John makes his move, then Nick makes a move again and so on. The player who takes the last pebble is the winner of the game.

a) What strategy can you offer Nick to win the game?

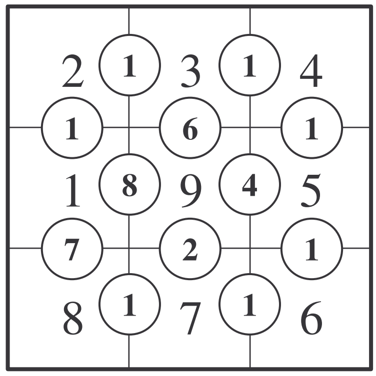
b) Can you offer John, as the second player, such a strategy?
(Give reasons for your answers)

[8]

Question 11

The numbers 1 to 9 must be placed in the squares on the grid. The numbers in the circles are the positive differences between the numbers in adjacent squares.
S is the sum of the numbers in the circles.
An example is given.

Example



S = 34

a) Show how to arrange the numbers 1 to 9 in the squares so that S is a maximum.

b) Explain clearly why no other arrangement could give a larger total than yours.

Use the diagram on the Answer Sheet to complete the above question

[8]

Question 12

Place algebraic operations $+$; $-$; $\div$ or $\times$ between the numbers 1 to 9 **in that order** so that the total equals 100. You may also freely use brackets before or after any of the digits in the expression and numbers may be placed together, such as 123 and 67 (see example).
Two examples are given below:

i) $123+45-67+8-9=100$

ii) $1+[(2+3)\times 4\times 5]-[(6-7)\times(8-9)]=100$

Four solutions will be awarded 2 marks each. Any other solution will get a bonus of 1 mark each to a maximum of 3 bonus marks.

[8]

Question 13

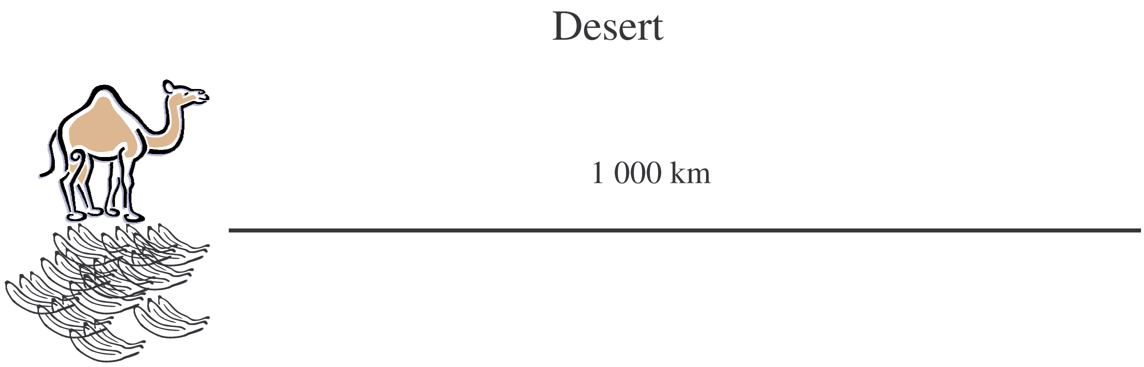
(a) Find three different positive integers, the sum of any two of which is a perfect square. (3)

(b) Find a general formula that will help generate other such triplets. (5)

[8]

Question 14

An age-old problem states the following:
A camel sits next to a pile of 3 600 bananas at the edge of a desert. He has to get as many bananas as possible, across this desert which is 1 000 km wide. He can only carry a maximum of 1 200 bananas at any one time. To survive he has to eat one banana for each kilometer he travels.
What is the maximum number of bananas that he can get to the other side of the desert?



[8]

Question 15

a) Prove that $3^{28} + 7^{51}$ is not a prime number. (2)

b) Prove that $2^{2009} + 5^{2010}$ is not a prime number. (6)

[8]

Total: 100

THE END

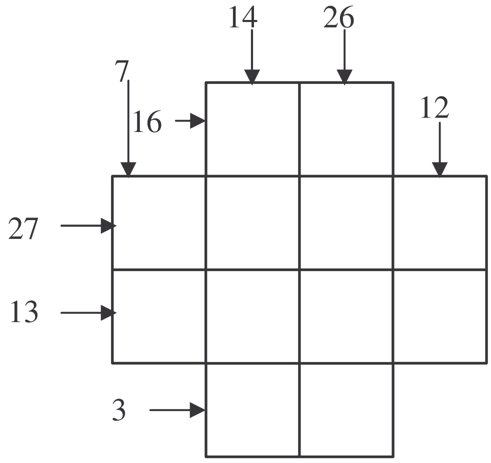
Please turn over for the answer sheet for questions 4, 5 and 11

ANSWER SHEET

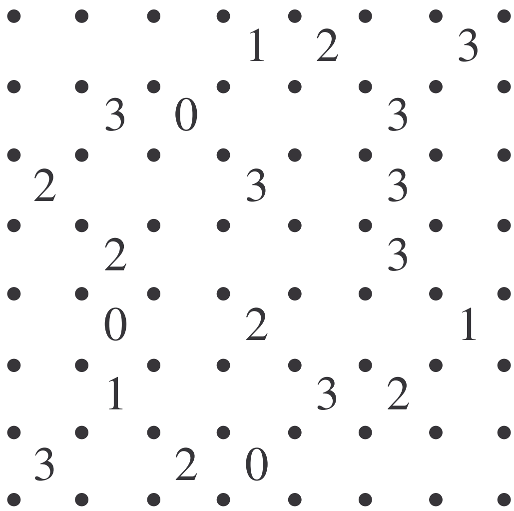
Please hand in together with your answer booklet.

Name: ______________________________ **School:** ________________________ **Grade:** ______

Question 4



Question 5



Question 11