HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

SAMF
SOUTH AFRICAN MATHEMATICS FOUNDATION

HARMONY™

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

THIRD ROUND 2008
JUNIOR SECTION: GRADES 8 AND 9

4 SEPTEMBER 2008
TIME: 4 HOURS
NUMBER OF QUESTIONS: 15

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- Questions 2, 4 and 5 should be done on the Answer Sheet provided – see last page.
  (Please remember to write your Name and School on the answer sheet)
- The neatness in your presentation of the solutions may be taken into account.
- Diagrams are not necessarily drawn to scale.
- No calculator of any form may be used.
- Answers and solutions are available at: www.samf.ac.za

DO NOT TURN THE PAGE
UNTIL YOU ARE TOLD TO DO SO.

PRIVATE BAG X173, PRETORIA, 0001
TEL: (012) 392-9323 FAX: (012) 320-1950
E-mail:ellie@samf.ac.za

Organizations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns

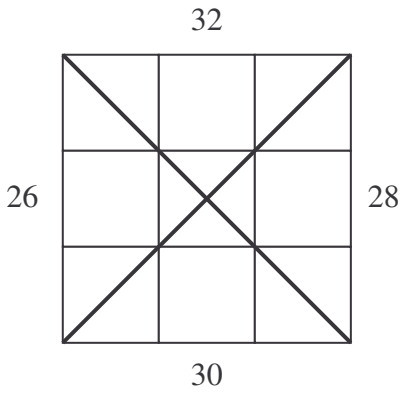
Question 1

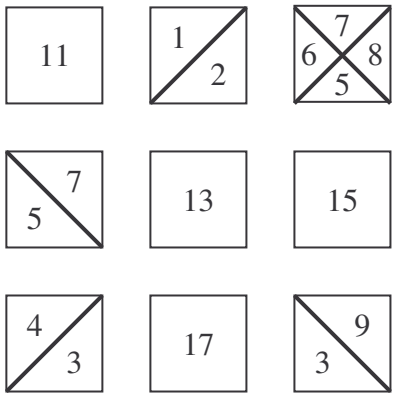
A teacher has to buy exactly 106 sweets. The sweets are sold in packs of 5 which cost R6 per pack, or packs of 7 which cost R7 per pack. What is the lowest cost at which the teacher can buy the sweets?

(4)

Question 2

Without rotating the small squares on the right, arrange them into the pattern shown in the diagram on the left, so that the number next to each large triangle equals the sum of the four numbers in that triangle.

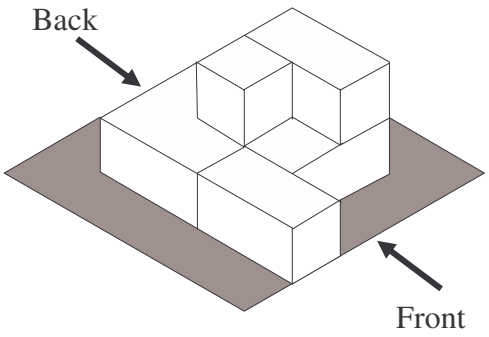


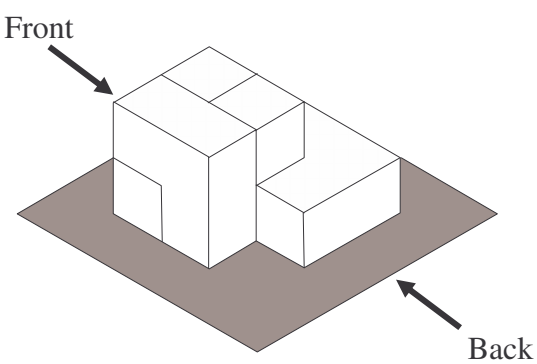


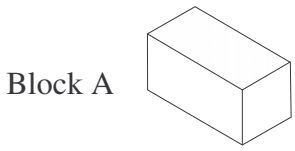
(4)

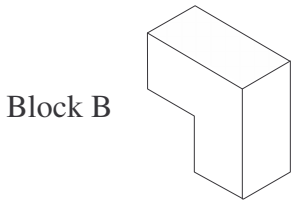
Question 3

The same pile of blocks is seen from 2 different sides. It is made from only two types of blocks (A and B). How many of each type are there in the pile?









(6)

Question 4

Draw a star (*) in 10 empty squares in the diagram alongside so that each numbered square accurately indicates how many immediately adjacent squares (horizontally, vertically or diagonally) contain a star.

				0		
				2		
						2
1						
		4			4	
		1	2	2	1	

An example is given below:

0	1	1
1	2	*
1	*	2

(6)

Question 5

In the faraway land of Mathopillis, along the south coastal road, lies a string of eight beautiful little villages. They are, in the order one would pass through them when travelling from West to East, Alpha, Beta, Circa, Dode, Epsilon, Flora, Gamma and Hexa.

The chart below indicates the distances in km between some of the villages. (For example, the distance between Alpha and Dode is 28km.) Find the distance between each village and the next one and complete the chart on the answer sheet.

Alpha							
	Beta						
		Circa					
28			Dode				
	27			Epsilon			
43		25			Flora		
			22			Gamma	
		38		24			Hexa

(6)

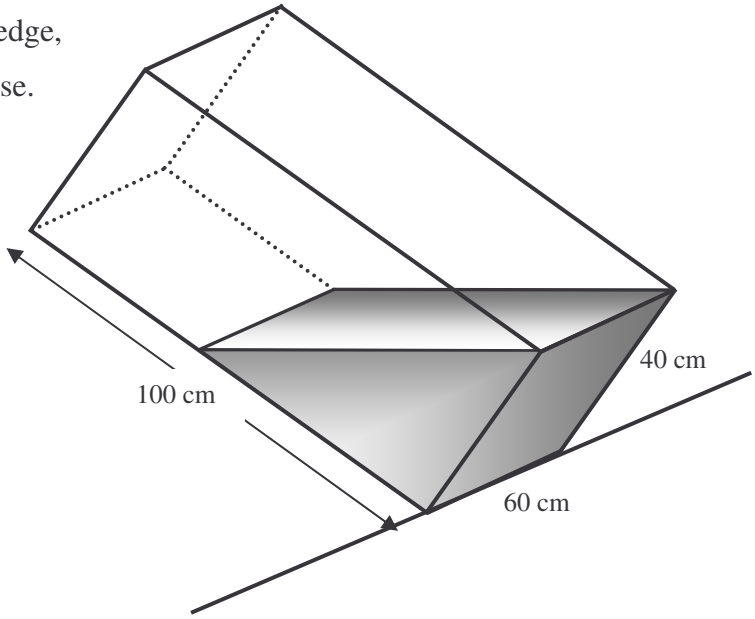
Question 6

On a wooden rod, there are markings for three different scales. The first set of markings divides the rod into 10 equal parts; the second set of markings divides the rod into 12 equal parts; the third set of markings divides the rod into 15 equal parts. If one cuts the rod at each marking, how many pieces of wood does one get?

(6)

Question 7

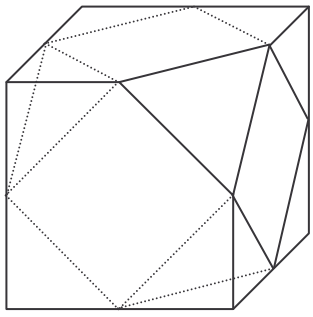
A fish tank is 100 cm long, 60 cm wide and 40 cm high.
If it is tilted, as shown, resting on the 60 cm edge, then the water reaches the midpoint of the base.
If it is then put down so that the base is horizontal again, what is the depth of the water?



(6)

Question 8

A cuboctohedron is a polyhedron that can be formed by slicing a cube at the midpoints of all its edges.
In the diagram, one of the vertices has been sliced off.
Find the surface area of the cuboctohedron formed from a cube having a side of length 4 cm.



(6)

Question 9

Find the length of the hypotenuse of a right-angled triangle in terms of its area A and its perimeter P.

(8)

Question 10

Find the smallest two digit number that satisfies the following conditions:

- The number is not an odd number.
- It has exactly four factors, including itself and 1.
- If you reverse the digits, a prime number is formed.
- The sum of the digits is a two digit prime number.
- One of the digits is a square number.

(8)

Question 11

Find the smallest integer that is divisible by all integers from 2 to 13 except for one pair of two consecutive integers in this range.

(8)

Question 12

Consider the following sequence in which t_1 is the first term, t_2 is the second term and t_n is the n^{th} term: $t_1 = 1; t_2 = 2$ and $t_n = \left(\frac{n-3}{n-1}\right) t_{n-2}$, where $n > 2$.

(a) Find t_{2007}.

(b) Find t_{2008}. **(8)**

Question 13

Observe that $39 = 3 \times 9 + 3 + 9$.

(a) Find all other two-digit numbers which are equal to the product of their digits plus the sum of their digits.

(b) Prove that there are no three-digit numbers which are equal to the product of their digits plus the sum of their digits.

(8)

Question 14

(a) Calculate: $\sqrt{4\times3\times2\times1+1}$.

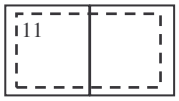
(b) Using the above, determine $\sqrt{51\times50\times49\times48+1}$ without actually multiplying out $51\times50\times49\times48$.

(c) Find and prove the general formula for the square root of the product of four consecutive integers plus 1.

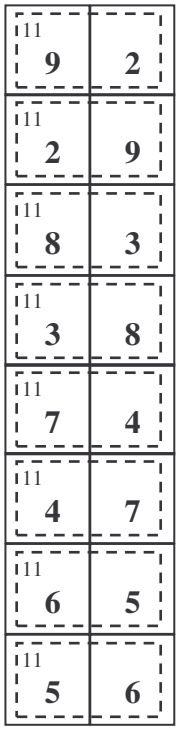
(8)

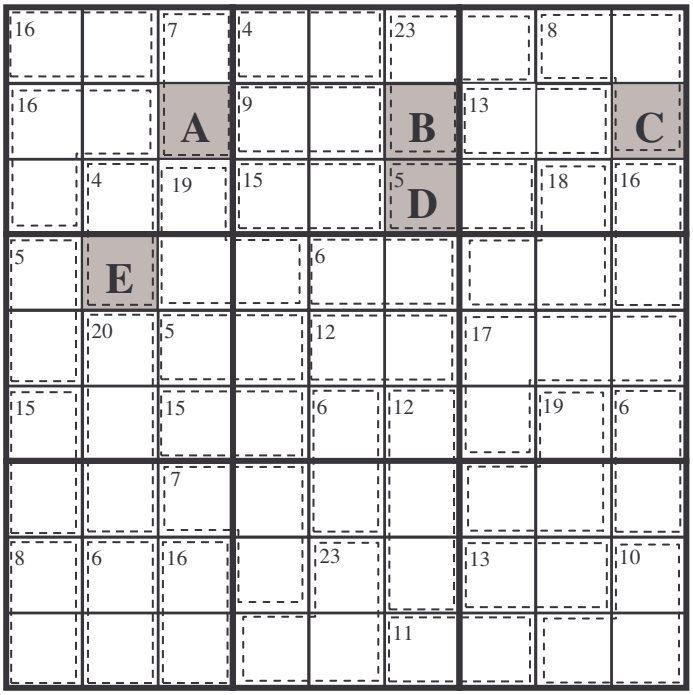
Question 15

In a killer Sudoku, just like in a conventional Sudoku, the aim is to fill each row, each column and each 3-by-3 block with all the numbers from 1 to 9. Your clues in a Killer Sudoku are the caged numbers that represent the sum of the numbers within that cage. Duplicate numbers cannot exist within a cage.

Keep in mind that 

could be any of the following combinations:





In the Sudoku above find the numbers represented by A, B, C, D and E.

(8)

Total: 100

THE END

Please turn over for the answer sheet for questions 2, 4 and 5

ANSWER SHEET
FOR QUESTIONS 2, 4 AND 5

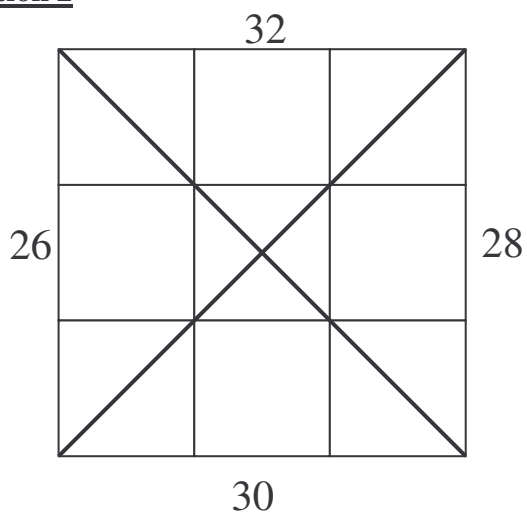
Please hand in together with Answer booklet.

Name: _____

School: _____

Grade: _____

Question 2



Question 4

				0		
				2		
						2
1						
		4			4	
		1	2	2	1	

Question 5

Alpha							
	Beta						
		Circa					
28			Dode				
	27			Epsilon			
43		25			Flora		
			22			Gamma	
		38		24			Hexa