SAMO 2010 – Junior First Round SOLUTIONS

1. B. $4\times5 + 3\times6 = 20 + 18 = 38$

2. D. Since 5 is a factor the number must end in either 5 or 0. Since there is no factor of 2, the last digit cannot be 0 so must be 5

3. C. $19 \div 11 = 1{,}72727\ldots..$ which uses 3 different digits

4. C. The prime possibilities are 23; 37; 53; 73

5. B. $45^2 = 2025$ and $2025 - 2010$ is 15

6. A. $5^{104} \times 4^{52} = 5^{104} \times 2^{104} = 10^{104} = 1000\ldots$ (i.e. 1 followed by 104 zeros) so sum of the digits is 1

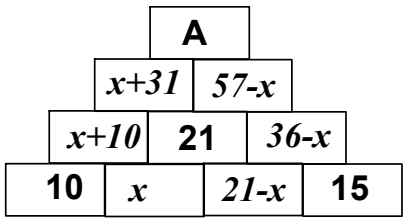
7. D. With the OR as one unit (in two ways) we have four things to arrange. So number of possibilities is $2 \times 4\times3\times2 = 48$

8. A. $$\frac{11!-9!}{11!+9!} = \frac{11.10.9!-9!}{11.10.9!+9!} = \frac{(11.10-1).9!}{(11.10+1)9!} = \frac{110-1}{110+1}$$

9. E. If the jug has capacity x ml then $80\% \times 60\% \times x = 192$.
So $0{,}48x = 192$ and $x = 192 \div 0{,}48 = 4 \times 48 \div 0{,}48 = 400$

10. A. PM = 1 unit. The shaded trapezium's area is
(average of parallel sides) × (distance between parallel sides)
$= (1 + 6)\times\frac{1}{2} \times 4 = 14$

11. A. Putting x into the left empty cell on the bottom, so that the other empty cell must contain $21-x$, we work upwards and then have $A = x + 31 + 57 - x = 88$.



12. A. $a = 6b$ and $4b = c$ and $a + c = 30$. Therefore $6b + 4b = 30$ and $b = 3$

13. E. Suppose the square starts with sides of length x cm. Because of the smaller squares being removed, the base of the box has sides of length $x - 2\times5$, and therefore the volume of the box is $5(x - 10)^2$. But then $(x - 10)^2 = 121$, so $x - 10 = 11$ and thus $x = 21$.

14. E. If the distance between Apetown and Beeville is x km, then the first speed is $\frac{x}{2}$ and the second speed is $\frac{x}{3{,}2}$. The difference between these is 30, so

$\frac{x}{2} - \frac{x}{3{,}2} = 30$, and thus $(3{,}2 - 2)x = 2 \times 3{,}2 \times 30 = 192$, so $x = 192 \div 1{,}2 = 160$.

OR

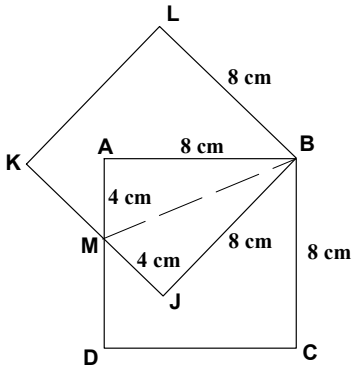
if v is the first speed, then we seek the value of $2v$, and we know that $3{,}2 \times (v - 30) = 2v$. This gives $v = 3{,}2 \times 30 \div 1{,}2 = 80$, so the required distance is 160 km.

15. B. BD + DC must exceed BC. Also DB + BC must exceed DC. Thus $10 > 2x + 1 > 4$. Then $\frac{9}{2} > x > \frac{3}{2}$, and since x is an integer, it must be 4 or 3 or 2, so sum of possible values is 9.

16. C.
Figure 1 = 4 matches = 1 × 4
Figure 2 = 10 matches = 2 × 5
Figure 3 = 18 matches = 3 × 6
Figure 4 = 28 matches = 4 × 7
…..
Figure 20 = 20 × (20 + 3) = 460

17. C. If the number of boys is x, the number of girls is $100 - x$.
Then considering A symbols gives $0{,}40x = 0{,}50(100 - x) + 4$,
so that $0{,}4x = 50 - 0{,}5x + 4$, giving $0{,}9x = 54$ and so $x = 60$.

18. C. Area of hexagon =
2 × (area of square) – 2 × area of Δ ABM)
= 2 × 8 × 8 – 2 × ½ × 4 × 8
= 128 – 32 = 96 cm^2



19. D. Let the even numbers be $2n$, $2(n\text{-}1)$, $2(n\text{-}2)$, …, $2(n\text{-}10)$.
Their sum is $(2n + 2n + 2n + \ldots.) - 2(1 + 2 + 3 + \ldots + 10)$
$= 11.2n - 2.\frac{1}{2}.10.11 = 11.2n - 11.10 = 11(2n - 10)$.

Since this is p, $2n - 10 = \frac{p}{11}$, and then $2n = \frac{p}{11} + 10$.

$1 + 2 + 3 + \ldots + k = \frac{1}{2}k(k+1)$ (see formula sheet)

OR

Let the even numbers be $m - 10$, $m - 8$, $m - 6$, …., $m + 6$, $m + 8$, $m + 10$.
Then their sum is $11m$ and is also p, so the largest is $\frac{p}{11} + 10$

20. C. Let A mean "Anne is telling the truth", $\overline{\text{B}}$ mean "Barbara is lying" etc.
Then either $\text{A} \Rightarrow \overline{\text{B}} \Rightarrow \text{C} \Rightarrow \overline{\text{D}}$ and two girls (Barbara and Diane) are lying
or $\overline{\text{A}} \Rightarrow \text{B} \Rightarrow \overline{\text{C}} \Rightarrow \text{D}$ and two girls (Anne and Catherine) are lying.