THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

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FIRST ROUND 2008
JUNIOR SECTION: GRADES 8 AND 9
ANSWERS AND SOLUTIONS

1. A	***11. B***
2. E	***12. C***
3. B	***13. B***
4. A	***14. D***
5. D	***15. C***
6. B	***16. D***
7. C	***17. C***
8. D	***18. C***
9. E	***19. D***
10.E	***20. D***

PRIVATE BAG X173, PRETORIA, 0001. TEL: (012) 392-9323
E-MAIL: ellie@samf.ac.za
Organisations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns

SOLUTIONS

1. **Answer A.** $0,4 \div 4 = 0,1$.

2. **Answer E.** $\frac{2008 \times 1710}{3421} \approx \frac{2000 \times 1700}{3400} = 1000$.

3. **Answer B.** $3\% \times R2008 + 7\% \times R2008 = (3\% + 7\%) \times R2008 = 10\% \times R2008 = R200,80$.

4. **Answer A.** We have

$$\begin{aligned} 2918 \times 149 &= 2918 \times (13278 - 13129) \\ &= 2918 \times 13278 - 2918 \times 13129 \\ &= 38745204 - 38310422 \\ &= 434782. \end{aligned}$$

5. **Answer D.** If a number is divisible by both 6 and 9, then it is divisible by 18 (the least common multiple of 6 and 9). The multiples of 18 between 100 and 500 are $6 \times 18 = 108,\ 7 \times 18 = 126,\ \ldots,\ 27 \times 18 = 486$. Hence, there are $27 - 6 + 1 = 22$ multiples of 18 between 100 and 500.

6. **Answer B.** There are $1 + 2 + 3 + 4 = 10$ triangles with height equal to the height of the biggest triangle, and there are $1 + 2 + 3 = 6$ triangles with height half that of the biggest triangle. Thus there are $10 + 6 = 16$ triangles in total.

7. **Answer C.** If the cost of a protractor is p and the cost of a set square is s, then $5p = 7s$. Hence $R62,00 = 5p + s = 7s + s = 8s$. Therefore, $4s = \frac{R62,00}{2} = R31,00$, and $28s = 7 \times (4s) = 7 \times R31,00 = R217,00$.

8. **Answer D.** We factorise both numbers: $150 = 2 \times 3 \times 5 \times 5$ and $105 = 3 \times 5 \times 7$. Thus the greatest common divisor of 150 and 105 is $3 \times 5 = 15$. There are at most 15 girls in the group.

9. **Answer E.** Since $10^2 = 100$ is just smaller than 101, the first square greater than 100 is $11^2 = 121$. The biggest square smaller than 300 is $17^2 = 289$. Thus there are $17 - 11 + 1 = 7$ squares between 101 and 300.

10. **Answer E.** The possible differences are:

$$\begin{aligned} (16 + 13) - (10 + 8) &= 29 - 18 = 11, \\ (16 + 10) - (13 + 8) &= 26 - 21 = 5 \text{ and} \\ (16 + 8) - (10 + 13) &= 24 - 23 = 1. \end{aligned}$$

Hence there are three possible answers.

11. **Answer B.** We express b, c and d in terms of a:

$$\begin{aligned}
b &= a+\tfrac{1}{5}, \\
c &= b+\tfrac{1}{6} = (a+\tfrac{1}{5})+\tfrac{1}{6} = a+\tfrac{11}{30}, \\
d &= a+\tfrac{1}{2}. \\
\text{Hence, } 9\tfrac{1}{15} &= a+b+c+d \\
&= a+(a+\tfrac{1}{5})+(a+\tfrac{11}{30})+(a+\tfrac{1}{2}) \\
&= 4a+\tfrac{6}{30}+\tfrac{11}{30}+\tfrac{15}{30} \\
&= 4a+\tfrac{32}{30} = 4a+1\tfrac{1}{15}. \\
\text{Thus, } 4a &= 9\tfrac{1}{15}-1\tfrac{1}{15} = 8 \\
\Longrightarrow a &= 2.
\end{aligned}$$

12. **Answer C.** Since Tickey and Sixpence collect money in the ratio 5:2, they collect $5x+2x=7x$ in total. But Tickey collects R600 more than Sixpence, so $R600 = 5x-2x=3x$, so $x=R200$ and they collect $7x = 7\times R200 = R1400$ in total.

13. **Answer B.** Let the two-digit number be $10x+y$. Then the three-digit number is $100x+50+y$, and the difference between the two numbers is $410 = (100x+50+y)-(10x+y) = 90x+50$. So $90x = 410-50 = 360$, which gives $x=4$.

    The sum of the digits of the three digit number is $x+5+y = 4+5+y = 9+y$ which equals 12, so $y = 12-9 = 3$. Thus, the difference between the digits is $4-3=1$.

14. **Answer D.** We need to find the value of $(399-397)+(395-393)+\cdots+(83-81)$. Each bracket equals 2, so we must find the number of brackets. The first number in each bracket is one less than a multiple of 4: $399 = 100\times4-1$, $395 = 99\times4-1$, $\ldots$, $83 = 21\times4-1$. Thus there are $100-21+1 = 80$ brackets, and so the sum is equal to $2\times80 = 160$.

15. **Answer C.** For the product to end in six zeroes, the product must be divisible by six factors of $10 = 2\times5$, so there must be six 2's and six 5's in the product. We have $16\times34\times75\times21\times13\times n = 2^4\times(2\times17)\times(3\times5^2)\times(3\times7)\times13\times n$, so there are $4+1=5$ factors of 2 and 2 factors of 5. Thus we need an extra factor 2 and four factor 5's. Thus the smallest value for n is $n = 2\times5^4 = 1250$.

16. **Answer D.**

$$\begin{aligned}
\sqrt{(1-\tfrac{1}{5})(1-\tfrac{1}{6})(1-\tfrac{1}{7})\ldots(1-\tfrac{1}{400})} &= \sqrt{(\tfrac{5-1}{5})(\tfrac{6-1}{6})(\tfrac{7-1}{7})\ldots(\tfrac{400-1}{400})} \\
&= \sqrt{(\tfrac{4}{\cancel{5}})(\tfrac{\cancel{5}}{\cancel{6}})(\tfrac{\cancel{6}}{\cancel{7}})\ldots(\tfrac{\cancel{399}}{400})} \\
&= \sqrt{\tfrac{4}{400}} = \sqrt{\tfrac{1}{100}} = \tfrac{1}{10}.
\end{aligned}$$

17. **Answer C.** If we arrange the tiles such that the sides of length 25cm is along the side of the floor of length 710 and the sides of length 20cm is along the side of the floor of length 430cm, then one can fit in $\frac{700}{25} = 28$ tiles along the 710cm side of the floor, and $\frac{420}{20} = 21$ tiles along the 430cm side of the floor, which gives a total of $28 \times 21 = 588$ tiles.

    If we arrange the tiles in the other way, then we can fit $\frac{700}{20} = 35$ tiles along the 710cm side of the floor, and $\frac{425}{25} = 17$ tiles along the 430cm side of the floor, which gives a total of $35 \times 17 = 595$ tiles.

    So the maximum number of tiles that can fit onto the floor is 595.

18. **Answer C.** The middle numbers forms the sequence $1,\ 5,\ 13,\ 25,\ \ldots$. Note that $1 = 0^2 + 1^2,\ 5 = 1^2 + 2^2,\ 13 = 2^2 + 3^2,\ 25 = 3^2 + 4^2,\ \ldots$. Thus the middle number of the 61st row is $60^2 + 61^2 = 3600 + 3721 = 7321$.

19. **Answer D.**

$$\begin{aligned} 65^2 &= 4225 \\ 665^2 &= 442225 \\ 6665^2 &= 44422225 \\ &\cdots \end{aligned}$$

    Thus, every time a 6 is added to the front of the number x, an extra 4 and an extra 2 are added onto the number x^2. Hence the sum of the digits increases by $4 + 2 = 6$ with each extra 6 added.

    So, since $\#[65^2] = 13 = 7 + 6$, $\#[665^2] = 19 = 7 + 2 \times 6$, if x has s sixes, then $\#[x^2] = 7 + 6s$. If $\#[x^2] = 49$, then $7 + 6s = 49$, which gives $6s = 49 - 7 = 42$. So $s = 7$, and $x = 66666665$.

20. **Answer D.** If you imagine folding the net into a cube, it is clear that that $\diamondsuit$ and $\heartsuit$ must always be on opposite faces of the cube, and similarly, ♣ and ☺ must be on opposite faces, as must ♠ and €. This rules out cubes (1), (2) and (4). Careful inspection shows that cubes (3) and (5) can indeed be obtained.